Pergamon

Mechanics Research Communications, Vol. 26, No. 1, pp. 65–68, 1999

0093-6413/99/$–see front matter

PII S0093-6413(98)00101-3

THE EXACT SOLUTION OF THE PLANE ELASTICITY PROBLEMS FOR THE S-CUT WITH CUSPS

E. A. Shirokova and P. N. Ivan'shin
Department of Mechanics and Mathematics, Kazan University,
Kazan, 420008, Russia

(Received 8 May 1998; accepted for print 25 August 1998)

Introduction

The rational function is found which maps the unit disk exterior on the plane with S-cut. The first and the second basic problem [1] are solved simultaneously.

Analysis

The method of solution of the problems is the same as in [2-4]. We find two functions

$$\Phi(\zeta) = \Gamma - \frac{X + iY}{2\pi(1+\kappa)z'(\infty)\zeta} + ..., \tag{1}$$

$$\Psi(\zeta) = \Gamma' + \frac{\kappa(X - iY)}{2\pi(1+\kappa)z'(\infty)\zeta} + ... \tag{2}$$

analytic in $E^- = \{\zeta = \xi + i\eta | |\zeta| > 1\}$ using the boundary condition

$$\left[k\Phi(\zeta)z'(\zeta) + \overline{\Phi(\zeta)}z'(\zeta) - z(\zeta)\overline{\Phi'(\zeta)}\zeta^{-2} - \overline{\Psi(\zeta)z'(\zeta)}\zeta^{-2}\right]\Big|_{\zeta=e^{i\theta}} = U(\theta) + iV(\theta). \tag{3}$$

Here $z(\zeta)$ is the function which maps E^- on the domain D - the plane with S-cut. This cut is not a slit over S. There are two cusps at the vertices of S and the edges of the cut diverge joining at the cusps tangentialy:

$$z(\zeta) = \zeta + \frac{(b^2-1)^2\zeta}{(\zeta^2 - b^2)(1+b^2)}, |b| < 1. \tag{4}$$